

# Risk Aware Net Load Balancing in Micro Grids with High DER Penetration

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**Abstract**—The rapid transformation of micro grids due to the accelerated integration of renewables, storage systems and IoT enabled monitoring and control has opened up new opportunities as well as challenges. Future micro grids will be characterized by high DER penetration and will require sophisticated net load balancing frameworks which explicitly consider the errors in prediction of load and generation due to uncertainty in weather conditions while making decisions. Traditional techniques for grid net load management which rely on isolated shaping of load and supply curves are inadequate and inefficient. For micro grids with high PV penetration, the intermittent and unpredictable nature of PV based energy generation can lead to dramatic and sudden supply demand imbalances thus requiring a holistic framework for balancing net load over the entire horizon. In this paper, we develop a sequential decision making framework for net load management that optimally balances the usage of storage and energy market transactions as a mechanism for mitigating supply demand imbalances (net load imbalances) over the horizon. Our framework specifically accounts for prediction uncertainty of future net load imbalances and minimizes the tail end risk of storage shortfall at the end of the horizon. Using qualitative analysis, we show that our framework achieves its objective of minimum cost net load balancing while accounting for the tail end risk.

**Keywords:** Net Load Balancing, Micro Grids, DER, MDP

## I. INTRODUCTION

Today's micro grids have large numbers of integrated IoT-enabled DERs (prosumers with storage systems, rooftop solar PVs). The falling cost of these components has enabled a dramatic increase in their installations. As per the DoE SunShot vision document, solar generated power is expected to grow to 14% of the total power supply in 2030 and 27% by 2050 [1]. Increasingly, energy storage systems are also being adopted to store surplus supply for future use. Advances in the efficiency and resiliency of energy storage systems [2] along with the proliferation of IoT enabled smart monitoring and control systems for the grid components [3] are opening up new challenges as well as opportunities for micro grid operations.

Renewable generation, due to its unpredictable nature, introduces uncertainty in net load balancing [4]. Traditional net load balancing frameworks for grids include load and supply

shaping techniques such as optimal power flow (OPF) based load distribution, market energy procurement, Security Constrained Economic Dispatch (SCED), etc. These techniques are usually performed in isolation. However, current micro grids with intermittent and highly uncertain PV generation require a more unified net load management framework.

We envision future micro grids to be separate monolithic components of the distribution system with interactions as single entities. Such micro grids will be characterized by high DER penetration, such as PV installations and energy storage systems. Each micro grid component will be a node in the IoT network enabling real time data collection and control. This large amount of IoT data will enable more frequent load and supply predictions, which can be advantageously integrated with decision making optimizations to develop more sophisticated net load balancing frameworks. Note that these frameworks should also account for prediction errors due to uncertainty in weather conditions.

In this paper, we develop a holistic sequential decision making framework for net load balancing over a short-term horizon in micro grids with high DER penetration. Given a set of load imbalance predictions for the future, the framework makes a set of actionable decisions over the horizon. Specifically, at each time interval, the framework decides the amount of (excess) energy to be bought (sold) from the market and discharged (charged) from the storage to mitigate current load imbalance while minimizing the total cost over the entire horizon. Our framework specifically accounts for prediction uncertainty of future net load imbalances and minimizes the tail end risk of storage shortfall at the end of the horizon.

## II. RELATED WORKS

Several works address the problem of mitigating net load imbalances by shaping the load/supply curve. Demand Response is a widely used technique which shapes the load curve by curtailing the load during peak demand periods [5], [6]. Techniques have also been developed focusing on directly controlling the load curve using electricity prices as a control [7]. To mitigate solar surplus, several techniques which perform solar curtailment have been developed [8], [9].

Several storage management techniques have also been developed to minimize the cost of grid operations. Techniques

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such as [10], [11] develop energy management systems which control storage.

Participating in electricity market by limiting the tail end risk was first proposed in the work done by Sethi et. al. [12]. Works such as [13], [14], [15] develop risk constrained framework for procuring wind energy. Works such as [16], [17] develop a similar framework for storage management under demand uncertainty. Our framework is similar to the framework in [17] as both perform storage management and energy market transactions simultaneously. However, Our MDP based solution allows us to perform net load balancing in the presence of discrete operational states [9]. The tail end risk that we consider is the unavailability of enough storage for the night time operations.

The techniques mentioned above work in isolation. Therefore, we develop a holistic framework that performs energy storage management and energy market transactions simultaneously allowing the grid operator to have a unified view of the grid.

### III. NET LOAD BALANCING MODEL

#### A. Micro Grid Model

The micro grid model that we consider in this work best represents a University or Industrial campus. It is characterized by the presence of several load consuming nodes such as buildings, and several producers of non-commercial scale (cheap) electricity. In this work, we assume that the producers supply electricity to the micro grid using PV installations over the buildings. The micro grid consists of distributed storage with an aggregate capacity  $R$ . The storage has a cumulative charge/discharge capacity of  $c$ . We do not model storage specific parameters such as Depth of Discharge (DoD), efficiency as we assume the presence of a number of distributed storage systems rather than a single battery system. A centralized controller is responsible for ensuring smooth grid operations over a decision making horizon defined over a set of daytime intervals  $t \in \{1, \dots, T\}$ . Note that the horizon is defined over daytime since supply within the micro grid using PVs is only available during that time. We assume that during the night time, all demand needs to be met using the storage accumulated during the day. In our framework, the controller responds to per interval supply demand mismatch by either procuring/selling electricity from the external market or by charging/discharging the storage. For each interval  $t$ , a buy action  $b_t$  needs to be performed, where  $b_t$  is the amount of electricity bought from the market (in kWh). Moreover, a discharge action  $sr_t$  is performed, where  $sr_t$  is the amount of the electricity discharged from the storage (in kWh). If  $b_t$  and  $sr_t$  are negative, opposite actions of selling the electricity or charging the storage, respectively are performed. The action performed at  $t$  is represented using  $a_t = \langle b_t, sr_t \rangle$ . The cost of buying (selling) electricity at interval  $t$  is  $C_t^b$ . We do not associate any cost with storage charging/discharging. We also assume that the set of costs  $C_t^b$ ,  $0 \leq t \leq T$  is known a priori.

#### B. Input Data Model

The inputs to our model are the load and generation predictions. As the predictions are prone to errors, they need to be modeled carefully so that the framework can make decision accounting for the errors. We assume that at the beginning of each interval  $t$  of the decision making horizon, the controller receives an input data vector  $\bar{Y}_t = \langle D_t, D_{t+1}, \dots, D_T \rangle$ . Here  $D_{t'} = L_{t'} - P_{t'}$ ,  $t' \in \{t, \dots, T\}$  denotes the per interval net load imbalance;  $L_{t'}$  and  $P_{t'}$  denote the per interval aggregate consumption and PV generation respectively.

At time  $t$ ,  $D_t$  denotes the actual net load imbalance as observed in real time whereas the  $D_{t'}$ ,  $t' \in \{t+1, \dots, T\}$  are random variables denoting the predicted imbalances. The relation between action  $a_t$  and  $D_t$  is  $D_t = b_t + sr_t$ . Thus, if we want to buy excess storage for the future at time  $t$ , we can choose  $b_t > D_t$ .

#### C. Tail End Risk

At the end of the decision making horizon, we assume that the minimum amount of storage required for night time operations is  $R'$ . Hence, the tail end event whose risk needs to be minimized is the unavailability of storage capacity of more than  $R'$  by the end of the decision making horizon. Depending upon the severity of the implications of failing to avoid the risk, the grid operator can associate it with a cost function as follows:

$$C(R_T, R') = \begin{cases} 0 & R_T \geq R' \\ C_{risk} & \text{otherwise} \end{cases} \quad (1)$$

where  $R_T$  denotes the storage available after the interval  $T$  and  $C_{risk}$  denotes the cost of not avoiding the risk.  $C_{risk}$  needs to be calculated appropriately to provide the operator with a choice to meet or not to meet  $R'$  for a given input data. In this work, we assume that the risk needs to be avoided at all cost and hence  $C_{risk} = \infty$ . We will extend our framework to handle partial risk tolerance in our future works.

#### D. Net Load Balancing Framework

A high level overview of our net load balancing framework is shown in Figure 1. A storage capacity of  $R_0$  is available at the beginning of the decision making horizon. In each time interval  $t$ , using the most updated input data  $\bar{Y}_t$ , the framework makes a decision and outputs action  $a_t$  for buying (selling) from the external market and discharging (charging) the storage. The action  $a_t$  for each interval is determined by formulating and solving a Markov Decision Process (MDP) [18] as described in the next section. MDP is a widely used mathematical framework for solving sequential decision making problems. At the end of the horizon, the amount of storage available  $R_T$  should be  $\geq R'$ .

### IV. RISK AWARE NET LOAD BALANCING FRAMEWORK

In this section, we discuss the details of the Risk Aware Net Load Balancing Framework that we developed to perform minimum cost per interval net load balancing using storage and electricity market while ensuring that the tail end risk of the shortfall of storage is avoided.

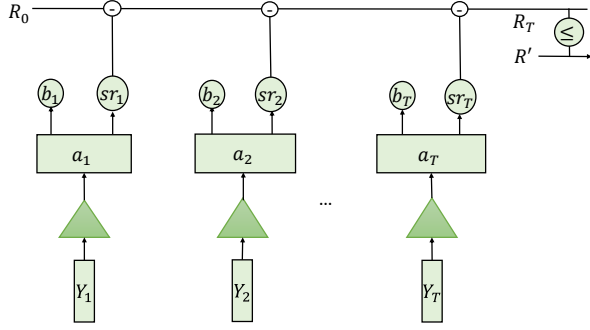


Fig. 1. Net Load Balancing Framework

### A. Objective

Given the net load balancing model defined in Section III, the objective of our framework is to minimize the cost of grid operations while ensuring that in each interval, the power ingress is equal to the power egress and at the end of the horizon, the minimum storage requirement is met.

### B. Solution

1) *MDP Formulation*: For each interval  $t \in \{1, \dots, T\}$ , with information  $\bar{Y}_t$ , we define the MDP to determine action  $a_t$  using the following parameters:

a) *Decision Epoch*: The decision epochs – the time intervals during which the MDP makes decisions – are  $\{t, \dots, T\}$  with  $T < \infty$ .

b) *States*: MDP uses the information regarding the current state to make a decision. For each interval  $t' \in \{t, \dots, T\}$ , we define state  $s_{t'} = \langle d_{t'}, R_{t'} \rangle$ , where  $d_{t'} \sim D_{t'}$  denotes the predicted net load imbalances.  $R_{t'}$ ,  $0 \leq R_{t'} \leq R$  denotes the available storage capacity and is defined as  $R_{t'} = R_{t'-1} - sr_{t'}$ .

c) *Initial and Terminal States*: The initial state is denoted using a single state  $s_{t-1} = \langle 0, R_{t-1} \rangle$ . The terminal states are denoted using  $s_T = \langle 0, R_T \rangle$  where  $R_T \geq R$ . Note that the net load imbalance for initial and terminal states are zero. This is because the initial state imbalance is addressed before invoking the current MDP and the terminal state is outside the decision epoch.

d) *Actions*: MDP outputs an action from all available actions in the action space for each time interval after making a decision. The action ensures that the load imbalance for the interval is mitigated while making sure that the storage charging/discharging constraints are satisfied. For a state  $s_{t'} = \langle d_{t'}, R_{t'} \rangle$ , action space contains actions  $a_{t'} = \langle b_{t'}, sr_{t'} \rangle$ ,  $t' \in \{t, \dots, T\}$  such that  $d_{t'} = b_{t'} + sr_{t'}$ ,  $\max\{R_{t'} - R, -c\} \leq sr_{t'} \leq \min\{-R_{t'}, c\}$  where  $c$  denotes the per interval charge/discharge capacity of the storage system. Only the action  $a_{t'}$  for  $t' = t$  i.e., corresponding to the current interval is executed. The remaining actions are part

of the computation framework while solving the objective function.

e) *State Transition Probabilities*: Given state  $s_{t'} = \langle d_{t'}, R_{t'} \rangle$  at time  $t'$ , the transition probability for  $s_{t'+1} = \langle d_{t'+1}, R_{t'+1} \rangle$  is as follows:

$$p(s_{t'+1}|s_{t'}, a_{t'}) = \begin{cases} 0 & \text{if } R_{t'+1} \neq R_{t'} - sr_{t'} \\ P[d_{t'+1}|\bar{Y}_t] & \text{otherwise} \end{cases}$$

where  $P[d_{t'+1}|\bar{Y}_t]$  denotes the probability distribution of  $d_{t'+1} \sim D_{t'+1}$  under input data  $\bar{Y}_t$ .

f) *Objective*: The MDP solves the following objective problem:

$$\min E\left[\sum_{t'=t}^T C_{t'}^b \cdot b_{t'} + C(R_T, R')\right] \quad (2)$$

subject to the conditions defined above.

2) *MDP Solution*: When input data  $\bar{Y}_t$  is available at the beginning of interval  $t$ , the MDP is used to determine the action  $a_t = \langle b_t, sr_t \rangle$  which are executed immediately. If the storage capacity at the beginning of the interval was  $R_{t-1}$ , then the storage capacity at the end of the interval  $R_t = R_{t-1} - sr_t$ . The MDP is solved as detailed below.

a) *Solving MDP*: We define the cost to go function at time  $t' \in \{t, \dots, T\}$  as follows:

$$J_{t'}(R_{t'}, d_{t'}) = C_{t'}^b \cdot (d_{t'} - sr_{t'}) + E[C(R_T, R') + \sum_{i=t'+1}^T C_i^b \times (D_i - sr_i)|\bar{Y}_t] \quad (3)$$

If  $J_{t'}^*$  is the optimal value of the function, then it can be written recursively as:

$$J_{t'}^*(R_{t'}, d_{t'}) = \inf_{sr \in \mathcal{SR}} \{C_{t'}^b \cdot (d_{t'} - sr_{t'}) + \sum_{d \sim D_{t'+1}} P[d|\bar{Y}_t] \times J_{t'+1}^*(R_{t'} - sr_{t'}, d)\} \quad (4)$$

where  $\mathcal{SR} = \{sr : \max\{R_t - R, -c\} \leq sr_t \leq \min\{R_t, c\}\}$ . The recurrence relation above implies that at time  $t'$  the best course of action is the one which minimizes the sum of the cost of operation in the current interval and the expected value of the future cost of operations. The above recurrence can be solved using standard dynamic programming techniques. The initial condition for the recurrence relation is as follows:

$$J_{T+1}^*(R_T, 0) = \begin{cases} 0 & \text{if } R_T \geq R \\ \infty & \text{otherwise} \end{cases} \quad (5)$$

The optimal action at time  $t$  can be determined by solving  $J_t(R_{t-1}, 0)$  after setting up the dynamic program for the MDP. The variables need to be discretized in order to solve the dynamic program.

b) *Runtime*: Let  $[a, b]_\delta$  denote the number of discrete entries when the range  $[a, b]$  is discretized using  $\delta$  units i.e.  $[a, b]_\delta = \lceil \frac{b-a}{\delta} \rceil$ . The total number of entries to be filled are:  $T \times [0, R]_\delta \times \max_t \{[d_t^{min}, d_t^{max}]_\delta\}$ , where  $d_t^{min/max}$  denote the minimum/maximum value attained by the random variable  $D_t$ . Each entry requires  $O(|\mathcal{SR}_\delta| \times \max_t \{[d_t^{min}, d_t^{max}]_\delta\})$  time. Hence, the total time complexity is  $O(T \times [0, R]_\delta \times |\mathcal{SR}_\delta| \times \max_t \{[d_t^{min}, d_t^{max}]_\delta\}^2)$ .

## V. EVALUATION

We evaluated the framework developed in Section IV to qualitatively assess its performance in terms of cost minimization and scalability. The framework was implemented using C++ on Dell Optiplex with 4 cores running at 2 Ghz clock frequency. The framework was run over a decision making horizon of 20 15-intervals (11 am - 4 pm).

### A. Dataset

We used electricity consumption (load) time series data available from USC’s micro-grid [19]. We selected a subset of buildings such that the average load during peak periods (1-4 pm) was 90 kWh and during off peak periods was 70 kWh. We assumed a storage capacity of 1500 kWh with a charge/discharge rate of 140 kW. The storage requirement at the end of the horizon was set as 1400 kWh, which is sufficient to supply off peak average load for 19 hours i.e. outside the decision making horizon. We used solar irradiance data available at [20] for Los Angeles USC Downtown area to calculate the solar generation data. We varied the area of solar panels and used PV output calculator [21] to generate three time series solar generation data which were then used to create three load imbalance time series. The three time series differed in the fraction of times either load or solar generation was surplus during the decision making horizon. Figure 2 shows the time series data of load and solar generation used to create the load imbalance data. The three solar time series are as follows: S1) Both load and solar surplus almost equal number of times, S2) load is surplus most of the time, and S3) solar generation is surplus most of the time. We assumed additive errors using gaussian distribution with mean  $\mu = 0$  and variance  $\sigma^2 = 5, 10, 20$  i.e.,  $Pr[e_{t'} | \bar{Y}_t] \sim \mathcal{N}(0, (t' - t) \times \sigma^2)$ , such that  $D_{t'} = \hat{d}_{t'} + e_{t'}$ , with  $\hat{d}_{t'}$  denoting the actual value of load imbalance.

### B. Optimality

We compare the cost of operation across the entire decision making horizon, i.e. 11 am - 4pm, of our framework against an optimal framework which has the correct data for the entire horizon in advance. The optimal framework solves a single MDP for decision epochs  $\{1, \dots, T\}$  using the available correct data. The cost of buying was set at \$0.75/kWh for 11am - 1pm and \$0.95/kWh for the remaining horizon. We evaluated our framework using 6 timeseries data generated i.e. three load imbalance profiles with two different variance of 10 and 20. We set the initial value of storage to 0 and 750 kWh.

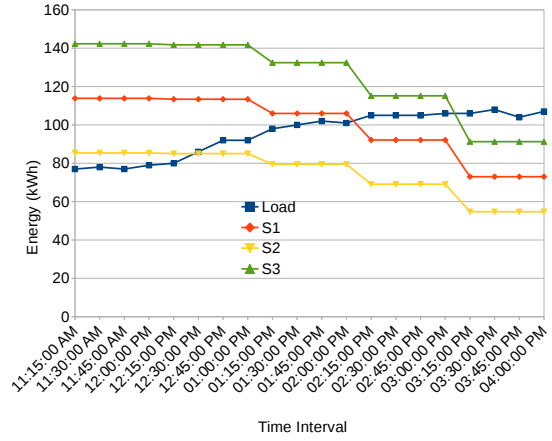


Fig. 2. Load and Solar Timeseries Data Used for Evaluation

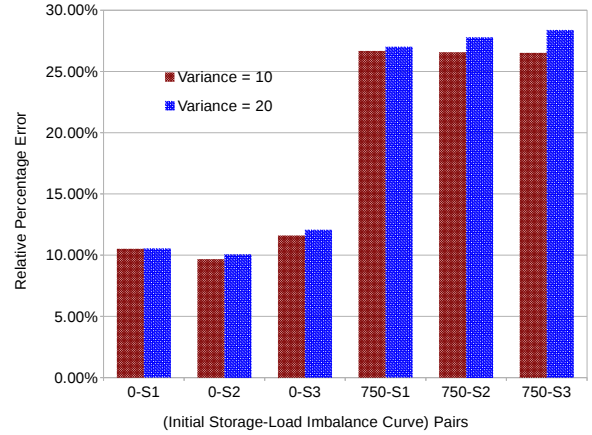


Fig. 3. Relative Percentage Error of Framework w.r.t. Optimal Cost For Varying Initial Storage and Variance

Figure 3 shows the relative percentage error of the cost of solutions obtained from our framework with respect to the optimal algorithm. As we can see from the figure, for each pair of initial storage-load imbalance curve, a higher variance leads to slightly higher errors. This is expected as increasing uncertainty in the input data increases the errors in decision making. However, note that the error difference is not significant implying that the framework is able to compensate for the increased uncertainty.

Also, note that the errors increase with the change in the initial storage value. This is due to the fact that when the initial storage is 0 kWh, the major focus of the framework is to just charge the storage to its final value (1400 kWh) and fulfill the deficit. However, when the initial storage is 750 kWh, charging requirements are reduced and the framework tries to focus on participating in the external market operations by selling the excess energy. The additional actions open up new avenues for the uncertainty in the input data to introduce errors as manifested in the higher relative percentage errors.

In our future work, we will focus on reducing the errors further by using reinforcement learning based techniques which learn from the past experience to improve the accuracy for the future operations.

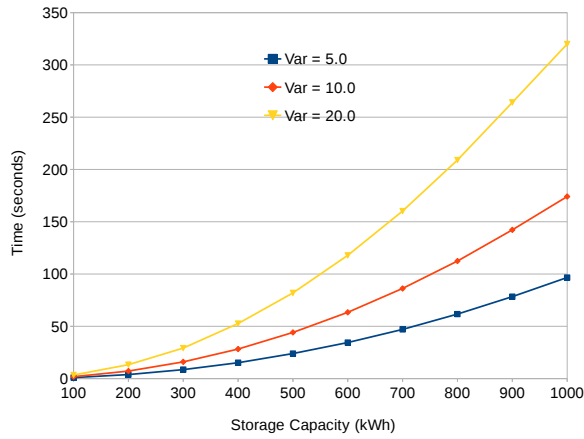


Fig. 4. Time Required for Decision Making w.r.t Storage Capacity for Varying Variance

### C. Scalability

We perform scalability analysis of the framework by varying the storage capacity and the variance of the errors. We calculate the time required to make decision at time period 0 for the entire horizon. For each storage capacity value  $R$ , we fix the charge discharge rate to  $0.1 \times R$ . Figure 4 shows the time required to make decisions with respect to the storage capacity  $R$  for various values of variance. The increase in runtime is quadratic in  $R$ . This is in agreement with the runtime which is  $O(R \times |\mathcal{SR}|)$ , when other variables are fixed and  $|\mathcal{SR}| = O(R)$ . The increase in runtime with variance is significant. However, even for a storage capacity of 1000 kWh and variance of 20, the runtime is just around 340 seconds.

A few techniques can be followed to further improve the scalability. The entries of the dynamic table have low interdependencies. Hence, they are easy to parallelize. Moreover, as the runtime is significantly affected by the storage capacity, the smart grid can be partitioned into several smaller grids with smaller storage capacity. This improvement in runtime would be quadratic in nature. As the focus of this paper is to develop a new net load balancing framework with reasonable performance, a detailed analysis of such techniques is omitted.

## VI. CONCLUSION

In order to sustain the current pace of Distributed Energy Resource (DER) integration into the smart grid, the existing energy infrastructure needs to be re-imagined. The framework developed in this work is one such effort in this regard. Our holistic framework which performs net load balancing using energy procurement and storage will allow the micro grid to have greater control over its operations. With a sufficiently high penetration of solar PVs, we envisage the micro grid becoming a self sustaining entity with minimal interaction with the larger grid.

In our future work, we will focus on data driven modeling of error probabilities to make our framework more suitable for real world deployment. We will also focus on modeling the

tail end risk to address scenarios other than the one discussed in this paper where it is avoided altogether.

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